## Appendix 3

## Arithmetic and Geometric Returns

### A. Introduction

One of the most important assumptions an actuary uses in measuring pension obligations is the discount rate. The exposure draft of ASOP No. 27 issued in January 2011 included the following question in transmittal memorandum:

> "4. Do you agree that the guidance on arithmetic and geometric returns is appropriate? Should the consequences of the use of geometric or arithmetic returns be disclosed?"

Given the wide range of responses received to the above question, the Pension Committee of the Actuarial Standards Board determined that the inclusion of some educational material regarding arithmetic and geometric returns in ASOP No. 27 would be beneficial. The following material is not meant to be an exhaustive discussion of the matter. It is meant to give the actuary some direction regarding the considerations that may be employed in determining whether the use of arithmetic or geometric returns is more appropriate in the selection of a discount rate. In many circumstances, as with the selection of other assumptions, the purpose of the measurement is one of the most important determinants.

The use of a *forward looking expected geometric return* as a discount rate will produce a present value that generally converges to the median present value as the time horizon lengthens (i.e., if the actuary determines a funding obligation using the *forward looking expected geometric return* to discount the obligation to produce a present value, it is expected that in the limiting case there will be enough money to fund the obligation 50% of the time). The use of a *forward looking expected arithmetic return* as a discount rate will generally produce a *mean* present value (i.e., there will be no expected actuarial gains and/or losses).

This appendix should not be construed as a preference for any particular present value measurements over others (for example, market-consistent present value measurements or measurements using a discount rate reflecting anticipated investment return).

### B. Looking Back Versus Looking Forward

The discount rate used in the measurement of a pension obligation is a forward-looking assumption. While the actuary may use some historical results in establishing expectations regarding the future, the discount rate reflects an expectation of events to come, not events that have already occurred.

One of the more confusing aspects of the debate regarding arithmetic and geometric returns is as follows:

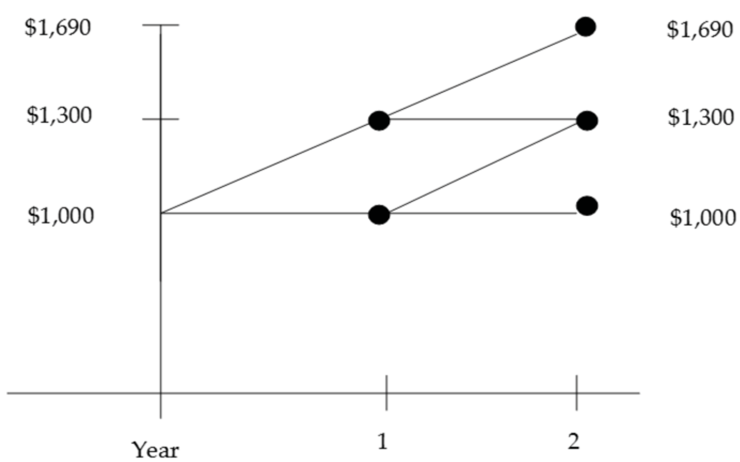
(a) determining whether we are talking about using historical results to establish forward looking (i.e., future) expectations, or

(b) determining whether we are talking about whether a *forward looking expected geometric return* or *forward looking expected arithmetic return* is a more appropriate discount rate

Note that a *forward looking expected geometric return* is not synonymous with compounding. That is, both a *forward looking expected geometric return* and a *forward looking expected arithmetic return* would be used in a compounding nature.

**C. An Example**

The following example illustrates the use of a *forward looking expected arithmetic return* to produce a *mean* present value. Assume that an asset class is expected to have a 50% probability of earning a return of 30% and a 50% probability of earning a return of 0% for each of the next two years and that these returns are the only possible outcomes. (The *forward looking expected arithmetic return* in this example would be 15%.) The chart below illustrates the totality of possible investment results for an initial $1,000 investment placed in this asset class:



The expected ending wealth values and a derivation of the *forward looking expected geometric return* is presented below:

| Ending Wealth | | | Rate of Return | | |
|---|---|---|---|---|---|
| \$1,690 x 1/4 | = | \$ 422.50 | $\left[\left(\frac{\$1,690}{\$1,000}\right)^{1/2} - 1\right]$ x 1/4 | = | 7.50% |
| \$1,300 x 2/4 | = | \$ 650.00 | $\left[\left(\frac{\$1,300}{\$1,000}\right)^{1/2} - 1\right]$ x 1/2 | = | 7.01% |
| \$1,000 x 1/4 | = | \$ 250.00 | $\left[\left(\frac{\$1,000}{\$1,000}\right)^{1/2} - 1\right]$ x 1/4 | = | 0.00% |
| Expected Value = | | \$1,322.50 | | | 14.51% |

The *forward looking expected geometric return* in this example is 14.51%. The question then becomes what discount rate would take the expected value of \$1,322.50 at the end of year 2 and produce a present value of \$1,000? The answer is shown below:

$$\text{Mean PV Rate of Return} = \left[\left(\frac{\$1,322.50}{\$1,000.00}\right)^{1/2} - 1\right] = 15\%$$

which is the *forward looking expected* *arithmetic* *return*. Note however in this simple example, that if the actuary funded an obligation that is expected to be \$1,322.50 at the end of year two with a one-time payment of \$1,000 at the beginning of year 1, there would be insufficient funds at the end of year 2 three-quarters of the time.

**D. Capital Market Assumptions from External Sources**

In many instances, the actuary will collect capital market assumptions from external sources in order to determine the *forward looking expected arithmetic return* and/or the *forward looking expected geometric return*. The capital market assumptions can be broadly classified into the following categories:

(a) expected returns by asset class;

(b) standard deviations by asset class; and

(c) correlation coefficients between asset classes.

With respect to expected returns by asset class, some external sources report *forward looking expected arithmetic returns*, some report *forward looking expected geometric returns* and some report both. It is important to understand what type of return was collected as well as the future time horizon to which the expected returns apply.

In general, a *forward looking expected geometric return* for an asset class can be approximated by taking the *forward looking expected arithmetic return* and subtracting one-half of the variance of the asset class[1].

If the actuary is trying to determine the *forward looking expected arithmetic return* for an entire portfolio from individual asset classes, this can be accomplished by taking the appropriate weightings from the individual asset classes' *forward looking expected arithmetic returns.* However, if the actuary is trying to determine the *forward looking expected geometric return* for an entire portfolio from individual asset classes, this cannot be accomplished by taking the appropriate weightings from the individual asset classes' *forward looking expected geometric returns.* In approximating the *forward looking expected geometric return* for the entire portfolio, the actuary would first determine the *forward looking expected arithmetic return* for the entire portfolio and then subtract one-half of the variance of the entire portfolio.

[1] Investments, Bodie, Kane and Marcus, 2005, p. 864.