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ASOP No. 27 Revision
Actuarial Standards Board (ASB)
1850 M Street, Suite 300
Washington, DC 20036

July 31, 2018

Subject: Proposed Revision of Actuarial Standard of Practice (ASOP) No. 27

To members of the Actuarial Standard Board:

I would like to thank the Actuarial Standards Board for the opportunity to provide comments on the proposed revision of ASOP No. 27. The focus of these comments is on the most problematic parts this ASOP:

- misguided guidance;
- misleading examples;
- misplaced creativity.

Misguided Guidance

Actuarial models are based on a substantial mathematical foundation. This foundation is presented in detail in actuarial textbooks, practice notes, and other educational materials. Consequently, ASOPs normally provides high level guidance without theoretical technicalities. Yet, "*… the Pension Committee of the Actuarial Standards Board determined that the inclusion of some educational material regarding arithmetic and geometric returns in ASOP No. 27 would be beneficial*." While this inclusion would be unusual, a summary of basic results supplemented by numerical examples and references to educational materials would certainly be beneficial.

The problem is there are no widely used educational materials in this area. A general closed form relationship between arithmetic and geometric returns is not known. Several known estimates of this relationship were scattered around miscellaneous publications. Despite the importance of this relationship, no single publication offered a systematic presentation of this relationship before 2011.

The apparent intention of the Pension Committee of the ASB was to produce a "first-of-its-kind" publication that would immediately become a standard. Producing such a publication would require conducting a comprehensive analysis of existing sources, researching analytical tools, and producing a solid justification for the results. Undoubtedly, this publication would be highly beneficial to the actuarial community and beyond. Undoubtedly, the committee was not properly equipped to produce this publication.

The final product of this endeavor published as Appendix 3 in the September 2013 version of ASOP No.27 is deeply flawed (this appendix is Appendix 2 in the current exposure draft). The current exposure draft contains several cases of misplaced creativity – misleading numerical examples, dubious recommendations, incorrect calculations, debatable statements, and other faults. Moreover, some substantive critical comments to the previous version of this ASOP were summarily ignored. Useful publications were evidently ignored as well.

This standard-setting fiasco demonstrates that emerging areas of actuarial practices should be developed in practice notes and other educational materials. As these areas evolve, practice notes and other publications would reflect this evolution. ASOPs should incorporate new areas only when these areas reach a certain level of maturity as related to general practices and the underlying theoretical foundation. Standard setters should refrain from pioneering new developments in ASOPs. These developments belong elsewhere.

Here are a couple of publications that may serve as a first step in the development of practice notes on the subject "arithmetic vs. geometric returns" (sent to the ASB in May 2012):

Mindlin (2011): On the Relationship between Arithmetic and Geometric Returns

Mindlin (2012): Present Values, Investment Returns and Discount Rates

I present just one example of the aforementioned faults in this section. Part D in Appendix 2 recommends the following approximation: "… *a forward-looking expected geometric return … can be approximated by taking the forward-looking expected arithmetic return and subtracting one-half of the variance …*" The authors seem unaware of other approximations of this kind. Mindlin (2011) presents three additional approximations and good reasons to believe that *the approximation recommended in the ASOP is the worst of the four*. Needless to say, dubious recommendations should have no place in an ASOP.

Later sections analyze certain problems in the current exposure draft in more detail.

Misleading Examples

Part C of Appendix 2 presents a numeric example that illustrates the relationship between arithmetic and geometric returns. This example has a couple of major problems.

It is assumed that the investor's portfolio "is is expected to have a 50% probability of earning a return of 30% and a 50% probability of earning a return of 0% for each of the next two years and that these returns are the only possible outcomes." This example deals with the following problem: *given $1,000 at the present, to calculate the distribution of accumulated asset values at the end of the investor's time horizon (in two years).*

Exhibit 1 in in part C of the appendix "illustrates the totality of possible investment results for an initial $1,000 investment" in two years.

***Exhibit 1*: Future Assets**

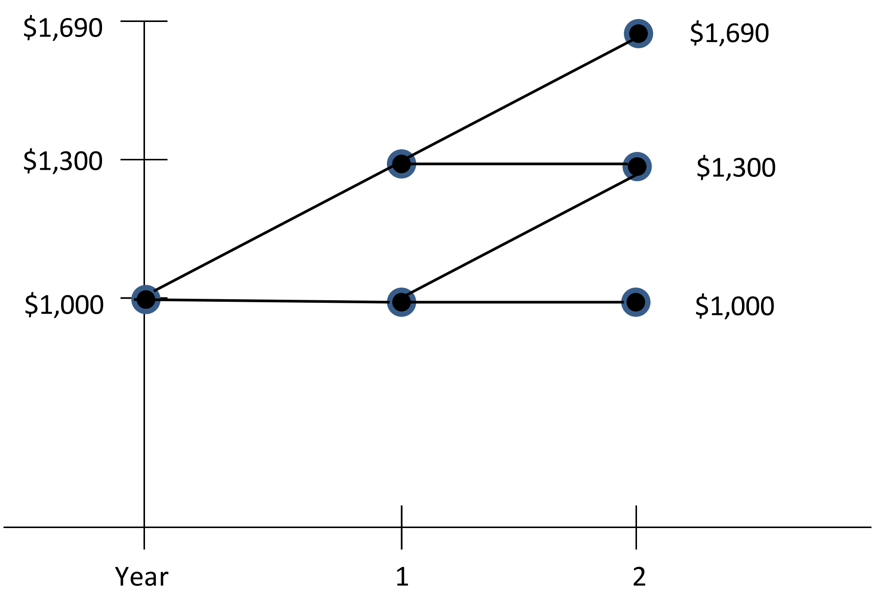


Exhibit 1 illustrates arguably the most important shortcoming of this example. The main task here is to calculate future values given present values. In contrast, the conventional problem of retirement funding is the opposite – to calculate present values given future values.

A long-established practice in actuarial science (and in many other areas of finance) is to measure the outcomes of investment programs at the present, not at the end of the investor's time horizon. *The choice of the measurement point in time may have profound consequences*. Actuarial valuations report present values, not future values. An example that does not represent long-established *actuarial* practices is not the best choice for an *actuarial* standard of practice. Context matters.

Another major problem with this example is the incorrect calculation of the geometric mean. “The forward-looking expected geometric return” (14.51%) not equal to the geometric mean of the portfolio in this example (see below). This is one of the aforementioned cases of misplaced creativity that should have no place in an ASOP.

Let us turn this example into a funding problem. The investor’s commitment is to accumulate $1,000 in two years. This example deals with the following problem: *to calculate the distribution of asset values at the present required to accumulate $1,000 in two years*. This example utilizes the same portfolio (equally probable returns of 30% and 0%).

The key measurements of the portfolio return – arithmetic mean *A*, geometric mean *G*, and variance *V* – are calculated as follows:

$$A = \frac{1}{2}30\% + \frac{1}{2}0\% = 15.00\%$$

$$G = exp\left(\frac{1}{2}ln(1 + 30\%) + \frac{1}{2}ln(1 + 0\%)\right) - 1 = 14.02\%$$ (see Mindlin (2011))

$$V = \frac{1}{2}(30\%)^2 + \frac{1}{2}(0\%)^2 - (15.00\%)^2 = 2.25\%$$

It is informative to test the approximation $G \approx A - \frac{1}{2}V$ currently recommended by this ASOP: $A - \frac{1}{2}V = 15.00\% - \frac{1}{2}2.25\% = 13.875\%$, which is lower than the geometric return 14.02%.

Fortunately, there is no need to use $G \approx A - \frac{1}{2}V$. The approximation $(1 + G)^2 \approx (1 + A)^2 - V$ should be expected to work better, and it is exact in this case:

$(1 + 14.02\%)^2 = (1 + 15.00\%)^2 - 2.25\%$ (see Mindlin (2011) for more details).

I question the wisdom of recommending a particular approximation in this ASOP.

Let us calculate the required assets distribution.

If the investment returns are 0% in the first year and 0% in the second year, then the required asset value is $1,000.00:

$$1{,}000.00 = \frac{1{,}000.00}{(1 + 0\%)(1 + 0\%)}$$

If the investment returns are 30% in the first year and 0% in the second year, then the required asset value is $769.23:

$$769.23 = \frac{1{,}000.00}{(1+30\%)(1+0\%)}$$

If the investment returns are 0% in the first year and 30% in the second year, then the required asset value is $769.23:

$$769.23 = \frac{1{,}000.00}{(1+0\%)(1+30\%)}$$

If the investment returns are 30% in the first year and 30% in the second year, then the required asset value is $591.72:

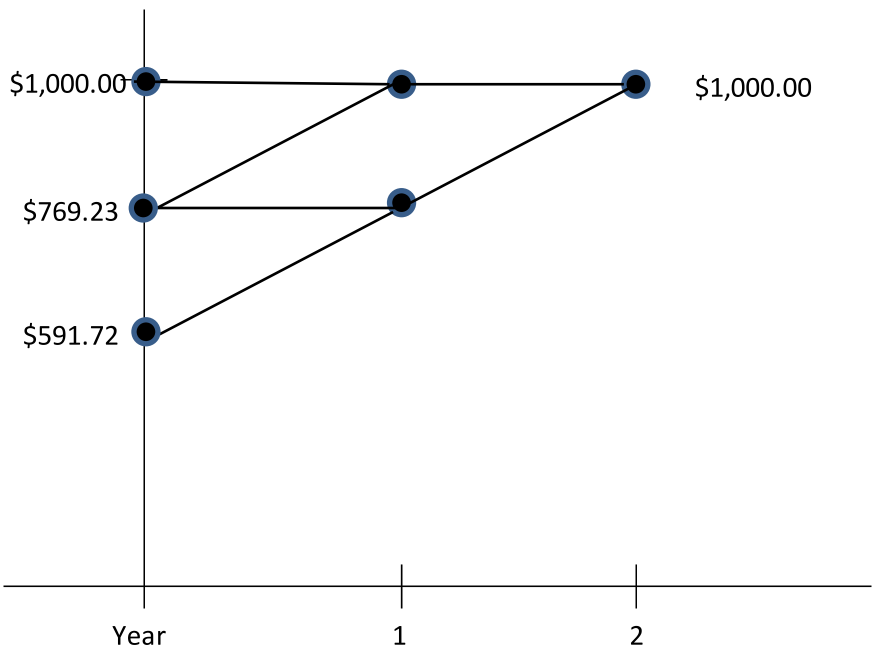
$$591.72 = \frac{1{,}000.00}{(1+30\%)(1+30\%)}$$

The required assets (*RA*) distribution is the following:

$1,000.00 with probability 25%;
$769.23 with probability 50%;
$591.72 with probability 25%.

Exhibit 2 below illustrates the required assets for the commitment of $1,000 in two years.

***Exhibit 2*: Required Assets**



The key measurements of required assets – the mean, median, and variance – are as follows:

$$RA\ Mean = 782.54$$
$$RA\ Median = 769.23$$
$$RA\ Variance = 21{,}014$$

If the actuary had to report a deterministic measurement of the required assets distribution, then the mean and the median would be reasonable choices (among others). The choice of the mean (782.54) as the deterministic measurement would imply discount rate 13.04%:

$$13.04\% = \left(\frac{1{,}000.00}{782.54}\right)^{\frac{1}{2}} - 1$$

The choice of the median (769.23) as the deterministic measurement would imply discount rate 14.02%:

$$14.02\% = \left(\frac{1{,}000.00}{769.23}\right)^{\frac{1}{2}} - 1$$

For comparison, let us calculate similar values for the original example. The future assets (*FA*) distribution is the following:

$1,000 with probability 25%;
$1,300 with probability 50%;
$1,690 with probability 25%.

The key measurements of future assets – the mean, median, and variance – are as follows:

$$FA\ Mean = 1{,}322.50$$
$$FA\ Median = 1{,}300.00$$
$$FA\ Variance = 60{,}019$$

If one had to present a deterministic measurement of the future assets distribution, then the mean and the median would be reasonable choices (among others). The choice of the mean (1,322.50) as the deterministic measurement would imply discount rate 15.00%:

$$15.00\% = \left(\frac{1{,}322.50}{1{,}000.00}\right)^{\frac{1}{2}} - 1$$

The choice of the median (1,300.00) as the deterministic measurement would imply discount rate 14.02%:

$$14.02\% = \left(\frac{1{,}300.00}{1{,}000.00}\right)^{\frac{1}{2}} - 1$$

These examples demonstrate the following:

- The outcomes of investment programs can be measured at the present or in the future.
- Present and future values are stochastic due to the investor's use of risky assets.
- Present and future value calculations do not require discount rates.
- The actuary may calculate present values first and, if necessary, determine discount rates next. Thus, the use of discount rates in present value calculations is a choice, not a necessity.
- The investor may want to select a deterministic measurement of outcomes that produces no expected gains or losses. The mean of outcomes at the measurement point would be a natural choice for this measurement. If the measurement point is at the present, then the implied discount rate is 13.04%. If the measurement point is at the end of the investor's time horizon, then the implied discount rate is 15.00%. Thus, the choice of the measurement point in time is highly consequential.
- The discount rates implied by the median present value and the median future value are the same and equal to the portfolio geometric mean (14.02%).
- The "no-gains-or-losses" discount rates for present values (13.04%) and future values (15.00%) are connected to the geometric mean (14.02%) via the following relationship:
  $$1 + 14.02\% = \sqrt{(1 + 13.04\%)(1 + 15.00\%)}$$

Overall, these examples should be largely re-written or eliminated altogether.

Misplaced Creativity

As was mentioned before, the current exposure draft contains several cases of misplaced creativity. The previous section contains one such case (the incorrect definition and calculation of the geometric mean). This section discusses a couple of such cases in this ASOP.

The first case involves the term "forward-looking expected arithmetic and geometric returns." Most publications define arithmetic and geometric *averages* for series of returns and arithmetic and geometric *means* (a.k.a. expected values, mathematical expectations, first moments, among other terms) for distributions of returns (random variables). See Mindlin (2011) for more details. The ASOP adds the modifier "forward-looking" to a known term and calculates the term incorrectly in a numeric example. To justify this addition, part B ("Looking Back Versus Looking Forward") of Appendix 2 contains the following paragraph:

"*The discount rate used in the measurement of a pension obligation is a forward-looking assumption. While the actuary may use some historical results in establishing expectations regarding the future, the discount rate reflects an expectation of events to come, not events that have already occurred*."

The problem with this correct statement is that it is equally correct regarding other actuarial assumptions (e.g. inflation rates, salary growth, mortality rates, etc.). Yet, there is no "forward-looking expected arithmetic and geometric" inflation rates in this ASOP. Quite appropriately, this ASOP utilizes just "inflation rates," even though the actuary "may use some historical results in establishing expectations regarding" the inflation rate assumption and distinguish arithmetic and geometric averages. It is understood that the inflation rate assumption "reflects an expectation of events to come, not events that have already occurred." Again, context matters.

This ASOP should eliminate "forward-looking expected arithmetic and geometric returns" and consistently utilize well-known arithmetic and geometric means instead.

The second case involve a perplexing paragraph in part A of Appendix 2:

"*The use of a forward-looking expected geometric return as a discount rate will produce a present value that generally converges to the median present value as the time horizon lengthens. ... The use of a forward-looking expected arithmetic return as a discount rate will generally produce a mean present value*."

These statements make no sense – mathematical or otherwise – and should be eliminated.

A somewhat similar statement is presented in part (j) "Arithmetic and Geometric Returns" of section 3.8.3:

"*The use of a forward-looking expected geometric return as an investment return assumption will produce an accumulated value that generally converges to the median accumulated value as the time horizon lengthens*."

This statement is mathematically suspect and should be eliminated.

I cannot help but mention the following striking paragraph in part B:

"*Note that a forward-looking expected geometric return is not synonymous with compounding. That is, both a forward-looking expected geometric return and a forward-looking expected arithmetic return would be used in a compounding nature*."

The meaning of this paragraph and the reasons for its inclusion in this ASOP elude this author.

Last but not least, this ASOP still provides no justification for the use of risk premium in "risk-free" rates of return (a.k.a. discount rates), even though this justification is readily available and would be greatly appreciated.

Conclusion

I believe that this exposure draft has significant room for improvement. As discussed in these comments, certain sections of the draft should be re-written or eliminated.

Thank you for your attention to these comments. Feel free to contact me if you have any questions/comments. I would be happy to assist the ASB in the development of this standard and related issues.

Sincerely

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