**COMMENT #1, MARCH 31, 2008**

**ASOP No. 27 Request for Comments**
**Selecting a Best-Estimate Range**
**Martin McCaulay, FSA, EA, MAAA**
**March 31, 2008**

*1. Under ASOP No. 27, an actuary selects an economic assumption by developing a "best-estimate range" and selecting a specific point within the best-estimate range. How do actuaries comply with the ASOP? What methodologies do they use to select a specific point within a "best-estimate range"? Is the "best-estimate range" approach the appropriate standard of practice? Does the ASOP inhibit the use of a more appropriate approach to selecting assumptions? Are there any specific changes that should be made to the ASOP to describe appropriate practice more accurately?*

Thank you for the opportunity to provide comments on ASOP No. 27. This paper describes the procedure for actuaries to select the best-estimate of the investment return range for a pension plan portfolio. Risk is measured using the standard deviation. Section 2.1 of ASOP 27 defines the best-estimate range as the narrowest range within which the actuary reasonably anticipates that the actual results, compounded over the measurement period, are more likely than not to fall. The long-term compounded annual rate of return can be determined from taking the expected arithmetic annual return and adjusting it for variance drain or volatility drag, which results in a reduction of about one-half of the variance[1].

Using the standard deviation of a portfolio, a best-estimate range of the long-term compounded rate of return is developed and expressed as a range between the 25th and 75th percentiles of the expected results. The range around the expected compounded return is plus or minus one-tenth of the standard deviation. The single point estimate is then selected from within this range.

**Approximate Best-Estimate Range of Annual Compounded Rate of Return =**
**Expected Annual Return – [Variance / 2] ± [Standard Deviation / 10]**

$$= \mu_p - .5\, \sigma_p^2 \pm [\sigma_p / 10]$$

For example, suppose a portfolio with mix of 50% domestic equities and 50% domestic fixed income has an expected arithmetic annual return of 7.50% and a standard deviation for the portfolio of 10%. The approximate expected annual compounded rate of return is $\mu_p - .5\sigma_p^2$, or 7.50% - [.5 (10%)$^2$], which is 7.00%. The approximate best-estimate range is $\mu_p - .5\sigma_p^2 \pm [\sigma_p / 10]$, or 7.00% ± [10% / 10], which is a range of 6.00% to 8.00%.

**Variance Drain**

[1] The variance is the standard deviation squared ($\sigma^2$).

The compounded return is lower than the expected annual return because of volatility. The variance drain is about one-half the variance of the portfolio[2]. For example, a

[2] MacBeth, James D. (1995). "What's the Long-Term Expected Return to Your Portfolio?" Financial Analysts Journal September/October 1995, Vol. 51, No. 5: 6-8995

sample portfolio of 100% equities with an annual expected rate of return of 10.0% and a standard deviation of 20% will have variance drain of about 200 basis points, and the compounded expected return is about 8.0%. Exact formulas for the compounded return after adjusting for the variance drain were developed by de La Grandville[3].

To improve on the estimate for the variance drain, a factor of 0.46 can be used instead of one-half. Using the 0.46 factor reproduces the results from the exact de La Grandville formula within about 2 basis points for portfolios with standard deviations under 17%. Standard deviations over 17% are found in portfolios with over 90% in equities, which is not common in pension funds.

**Annual Compounded Rate of Return =**
**Expected Annual Return – [0.46 (Variance)]**

$$= \mu_p - .46\,\sigma_p^2$$

**Rebalancing Premium**

A careful rebalancing process can add value by reducing volatility in a portfolio of weakly correlated securities[4]. The reduction from variance drain can theoretically be partially offset by rebalancing the portfolio. For the rebalancing premium to be significant, the portfolio assets need to have very low correlations.

**Expenses**

Investment expenses need to be subtracted from the expected return to produce the net return. Total expenses for defined benefit plans are typically about 30 basis points, or 0.3% of plan assets[5].

**Inter-quartile Range**

The best-estimate range of the expected return is the range in which the results are more likely than not expected to fall[6]. This is the range from the 25th to 75th percentile, or the inter-quartile range. The inter-quartile range should be constructed using the long-term expected standard deviation, which will decrease over time. The inter-quartile range for the 50-year compounded rate of return is the return plus or minus the standard deviation divided by ten.

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[3] de La Grandville, Olivier (1998). "The Long-Term Expected Rate of Return: Setting It Right." *Financial Analysts Journal*, Vol. 54:No. 6, 75-80

[4] Stein, David, PhD, Parametric Portfolio Associates, "Structuring Emerging Market Portfolios for Long-Term Growth", presentation to Texas Association of Public Employee Retirement Systems, March 15, 2008.

[5] Source: Center for Retirement Research at Boston College, "Why Have Some States Introduced Defined Contribution Plans?", January 2008.

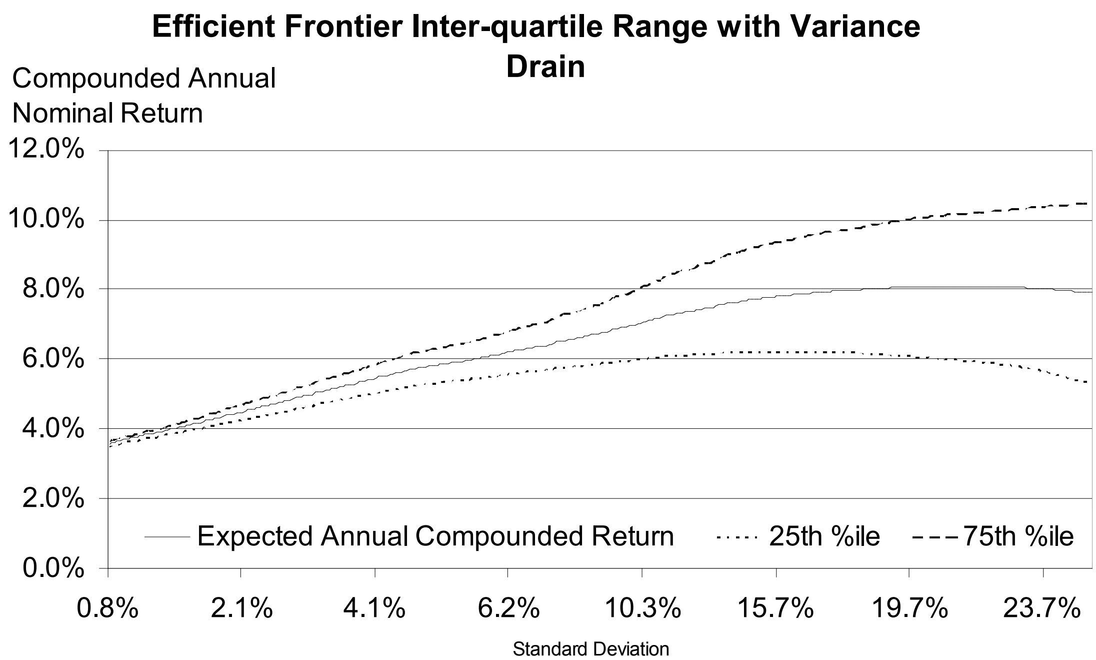
[6] Actuarial Standard of Practice No. 27 (ASOP 27), Selection of Economic Assumptions for Measuring Pension Obligations, Section 2.1.

There is a formula for the amount of the reduction in the standard deviation over time[7]. After 50 trials, the standard deviation will be reduced by dividing the original standard deviation by $50^{1/2}$. Using Chebyshev's theorem, the one-half of the results that fall in the inter-quartile range must be within $1/(2^{1/2})$ standard deviations of the mean[8]. The formula for the corridor for the inter-quartile range after 50 years is one divided by the product of $50^{1/2}$ times $2^{1/2}$, or one divided by $100^{1/2}$, and $100^{1/2}$ equals ten.

**Efficient Frontier**

The following graph is based on the Callan Associates 2008 capital market assumptions[9] which are shown in the appendix. The graph shows the efficient frontier after adjusting for variance drain, with the expected annual compound return on the x-axis and the standard deviation on the y-axis. The inter-quartile range is shown by using a corridor of one-tenth of the standard deviation around the expected compounded annual return.

The returns shown are before subtracting expenses. As the volatility increases, the compounded return peaks then starts to decrease, as the volatility drain from the extra risk reduces the expected return to be below that of less risky portfolios.




Note: Data created using 2008 capital market assumptions from Callan Associates, adjusted for variance drain.
Source: Texas Pension Review Board

7 Chuli, Roy M. Quantitative Analysis: An Introduction. CRC Press, 1999, p. 424
8 No more than $1/k^2$ of the values are more than $k$ standard deviations away from the mean.
9 Callan Associates provided their 2008 capital market assumptions to the Employees' Retirement System of Texas (ERS) on February 26, 2008. Callan Associates used an inflation assumption of 2.75%. The average CPI increase for 1927 through 2006 was 3.2%.

**Conclusion**

Based on the asset mix, the expected compounded return assumption before expenses can be developed by taking the expected annual return and subtracting about one-half, or 0.46, of the variance. The inter-quartile range can be developed by using a corridor within one-tenth of a standard deviation around the expected compounded return.

Rebalancing the portfolio can add a premium to the return if the assets have very low correlations. Expenses need to be subtracted from the compounded annual return to get the net investment return assumption.

**Best-Estimate Range of Annual Compounded Rate of Return =**
**Expected Annual Return – [Variance / 2] ± [Standard Deviation / 10]**

$$= \mu_p - .46\,\sigma_p^2 \pm [\sigma_p / 10]$$

The expected net annual compounded rate of return and best-estimate ranges for sample pension fund portfolios of 50% to 100% equities and 50% to 0% in fixed income investments are shown below, after subtracting assumed investment expenses of 30 basis points. For example, for a sample portfolio with 60% in equities and 40% in fixed income investments, with an annual expected return of 8.0% and a standard deviation of 12.0%, the net expected compounded return is in the 5.8% to 8.2% range, with a best-estimate of about 7.0%.

| Equities | Fixed Income | Annual Expected Return before Expenses | Standard Deviation | Expected Net Compounded Return | Best-estimate Range |
|---|---|---|---|---|---|
| 50% | 50% | 7.6% | 10.4% | 6.8% | 5.7% to 7.8% |
| 60% | 40% | 8.0% | 12.0% | 7.0% | 5.8% to 8.2% |
| 70% | 30% | 8.5% | 14.0% | 7.3% | 5.9% to 8.7% |
| 80% | 20% | 8.9% | 15.7% | 7.5% | 5.9% to 9.1% |
| 90% | 10% | 9.2% | 17.0% | 7.6% | 5.9% to 9.3% |
| 100% | 0% | 10.0% | 20.5% | 7.8% | 5.7% to 9.8% |

The expected net annual compounded rate of return for the sample pension fund portfolio of 100% equities and 0% fixed income investments, with an annual expected return of 10.0% and a standard deviation of 20.5% is in the 5.7% to 9.8% range, with a best-estimate of about 7.8%.

# Appendix

2008 Capital Market Assumptions from Callan Associates

| Asset Class | Annual Nominal Return | Annual Real Return | Standard Deviation | Yield |
|---|---|---|---|---|
| Broad Domestic Equity (Dom Eq) | 9.00% | 6.25% | 16.90% | 2.10% |
| Large Cap (Lg Cap) | 8.85% | 6.10% | 16.40% | 2.20% |
| Small/Mid Cap (Sm Mid Cap) | 9.85% | 7.10% | 22.70% | 1.20% |
| International Equity (Int Eq) | 9.00% | 6.25% | 19.20% | 2.00% |
| Emerging Markets Equity (EmMkt) | 9.60% | 6.85% | 31.20% | 0.00% |
| Domestic Fixed (Dom Fix) | 5.25% | 2.50% | 4.50% | 5.25% |
| Defensive (ST Fix) | 4.00% | 1.25% | 2.30% | 4.00% |
| TIPS | 4.90% | 2.15% | 6.00% | 4.90% |
| High Yield (High Yld) | 7.00% | 4.25% | 11.50% | 7.00% |
| Non US$ Fixed (Non US Fix) | 5.15% | 2.40% | 9.60% | 5.15% |
| Real Estate (RE) | 7.60% | 4.85% | 16.50% | 6.00% |
| Private Equity (Priv Eq) | 12.00% | 9.25% | 34.00% | 0.00% |
| Absolute Return (Abs Ret) | 6.50% | 3.75% | 9.70% | 0.00% |
| Cash Equivalents (Cash) | 3.50% | 0.75% | 0.80% | 3.50% |
| | | | | |
| Inflation | 2.75% | | 1.40% | |

------------- Correlation Coefficients ----------------------------

| | Dom Eq | Lg Cap | Sm Mid Cap | Int Eq | Em Mkt | Dom Fix | ST Fix | TIPS | High Yld | Non US Fix | RE | Priv Eq | Abs Ret | Cash |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. Dom Eq | 1.00 | 0.96 | 0.92 | 0.70 | 0.50 | 0.20 | 0.20 | -0.04 | 0.66 | -0.03 | 0.54 | 0.68 | 0.56 | -0.12 |
| 2. Lg Cap | 0.96 | 1.00 | 0.84 | 0.70 | 0.50 | 0.21 | 0.21 | -0.04 | 0.65 | -0.01 | 0.54 | 0.68 | 0.55 | -0.10 |
| 3. Sm Mid Cap | 0.92 | 0.84 | 1.00 | 0.63 | 0.44 | 0.14 | 0.14 | -0.05 | 0.59 | -0.06 | 0.47 | 0.62 | 0.52 | -0.15 |
| 4. Int Eq | 0.70 | 0.70 | 0.63 | 1.00 | 0.45 | 0.15 | 0.15 | -0.10 | 0.55 | 0.21 | 0.47 | 0.64 | 0.50 | -0.25 |
| 5. Em Mkt | 0.50 | 0.50 | 0.44 | 0.45 | 1.00 | 0.10 | 0.10 | -0.14 | 0.30 | -0.02 | 0.39 | 0.50 | 0.32 | -0.15 |
| 6. Dom Fix | 0.20 | 0.21 | 0.14 | 0.15 | 0.10 | 1.00 | 0.94 | 0.40 | 0.29 | 0.32 | 0.17 | 0.15 | 0.40 | 0.30 |
| 7. ST Fix | 0.20 | 0.21 | 0.14 | 0.15 | 0.10 | 0.94 | 1.00 | 0.40 | 0.29 | 0.32 | 0.17 | 0.15 | 0.40 | 0.35 |
| 8. TIPS | -0.04 | -0.04 | -0.05 | -0.10 | -0.14 | 0.40 | 0.40 | 1.00 | 0.15 | 0.11 | 0.00 | -0.03 | 0.05 | 0.29 |
| 9. High Yld | 0.66 | 0.65 | 0.59 | 0.55 | 0.30 | 0.29 | 0.29 | 0.15 | 1.00 | 0.10 | 0.55 | 0.47 | 0.45 | 0.07 |
| 10. Non US Fix | -0.03 | -0.01 | -0.06 | 0.21 | -0.02 | 0.32 | 0.32 | 0.11 | 0.10 | 1.00 | 0.03 | 0.10 | 0.15 | -0.05 |
| 11. RE | 0.54 | 0.54 | 0.47 | 0.47 | 0.39 | 0.17 | 0.17 | 0.00 | 0.55 | 0.03 | 1.00 | 0.44 | 0.40 | -0.06 |
| 12. Priv Eq | 0.68 | 0.68 | 0.62 | 0.64 | 0.50 | 0.15 | 0.15 | -0.03 | 0.47 | 0.10 | 0.44 | 1.00 | 0.43 | 0.07 |
| 13. Abs Ret | 0.56 | 0.55 | 0.52 | 0.50 | 0.32 | 0.40 | 0.40 | 0.05 | 0.45 | 0.15 | 0.40 | 0.43 | 1.00 | 0.50 |
| 14. Cash | -0.12 | -0.10 | -0.15 | -0.25 | -0.15 | 0.30 | 0.35 | 0.29 | 0.07 | -0.05 | -0.06 | 0.07 | 0.50 | 1.00 |

**Formulas for the Expected Annual Return and Standard Deviation**

For a given asset mix, the expected annual real rate of return ($\mu$) is the weighted average of the real returns of the included asset classes included.

$$\mu_p = w_1\ \mu_1 + w_2\ \mu_2$$

For a mix of 60% domestic equities and 40% domestic fixed income, with expected real returns of 6.25% for equities and 2.50% for fixed income, the expected annual real rate of return is 60% of 6.25% plus 40% of 2.50%, or 4.75%.

Expected annual real return: $\mu_p$ = 60%(6.25%) + 40%(2.50%) = 4.75%

Section 2.2 of ASOP 27 defines inflation as general economic inflation, or price changes over the whole of the economy. The assumed inflation of 2.75% is added to the assumed annual real rate of return to get a nominal assumed annual rate of 7.50%.

Expected annual nominal return: 4.75% + 2.75% = 7.50%

The formula for the standard deviation ($\sigma$) uses the weighted average of the variances for each class and a term that includes the correlation coefficient ($\rho$).

$$\sigma_p = [w_1\ \sigma_1^{\ 2} + w_2\ \sigma_2^{\ 2} + 2w_1w_2\rho_{12}\sigma_1\sigma_2]^{1/2}$$

For a mix of 60% domestic equities and 40% domestic fixed income, with standard deviations of 16.90% for equities and 4.50% for fixed income and a correlation coefficient of 0.2, the standard deviation for the portfolio is 13.67%.

$$\sigma_p = [60\%(16.90\%)^2 + 40\%(4.50\%)^2 + 2(60\%)(40\%)(0.2)(16.90\%)(4.50)]^{1/2} = 13.67\%$$

Standard deviation = 13.67%